Impulsive boundary value problems

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A little history about a cooperation with Milan and some new results

1995-1998

Common workshops in Olomouc and Katowice

1995-Workshop in Olomouc

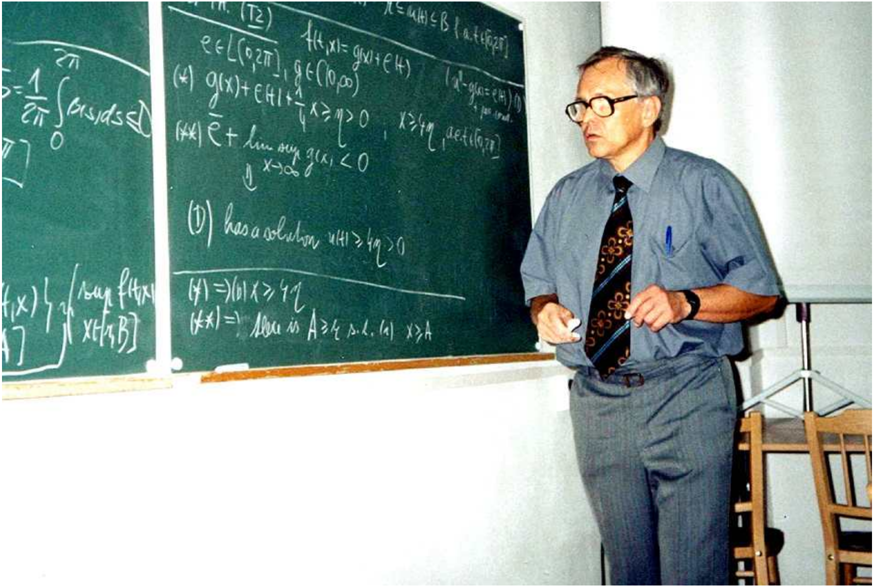
1996-Workshop in Katowice

1997-Workshop in Olomouc

1997-Workshop in Olomouc

1998-Workshop in Katowice

1998-Workshop in Katowice



1998-2000

Conferences in Ariel, Porto, Catania

1998-Conference in Ariel

1998-Conference in Ariel

1999-Conference in Porto

2000-Conference in Catania

Common papers 2000–2001

- Method of lower and upper functions and the existence of solutions to singular periodic problems for second order nonlinear differential equations, *Math. Notes Miskolc* 2 (2000), 135-143.

- Existence of nonnegative and nonpositive solutions for second order periodic boundary value problems, *Journal of Differential Equations* 176 (2001), 445-469.

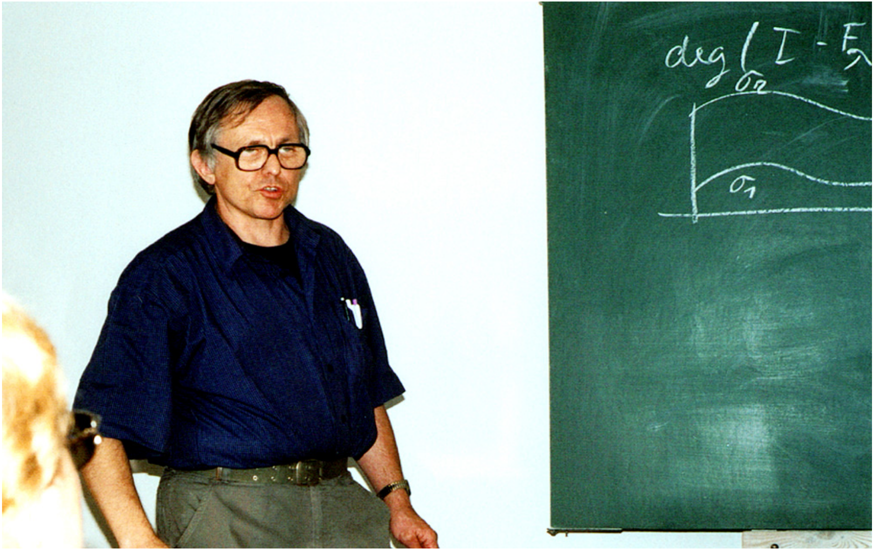
- Nonlinear systems of differential inequalities and solvability of certain nonlinear second order boundary value problems, *Journal Inequal. Appl.* 6 (2001), 199-226.

2001

Workshop in Olomouc

2001-Workshop in Olomouc

2001-Workshop in Olomouc



2001-Workshop in Olomouc

2001-2002

Conferences in Prague, London, Patras

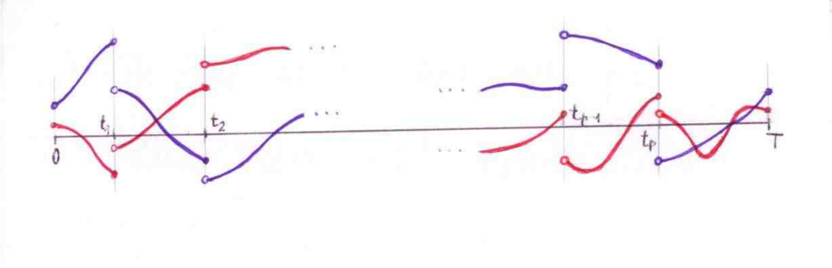
Fixed-time impulses

Consider $p \in \mathbb{N}$, $[0, T] \subset \mathbb{R}$ and the fixed points

$$0 = t_0 < t_1 < t_2 < \cdots < t_{p-1} < t_p < t_{p+1} = T.$$

Solutions are searched in the Banach space APC^1 with the norm

$$\|u\| = \|u\|_\infty + \|u'\|_\infty, \quad \text{where} \quad \|u\|_\infty = \sup_{t \in [0,T]} |u(t)|.$$



Periodic BVP with fixed-time impulses

We investigated the nonlinear periodic boundary value problem with fixed-time impulses

$$u''(t) = f(t, u(t), u'(t)), \tag{1}$$

$$u(t_i+) = J_i(u(t_i)), \quad u'(t_i+) = M_i(u'(t_i)), \quad i = 1, \dots, p, \tag{2}$$

$$u(0) = u(T), \quad u'(0) = u'(T), \tag{3}$$

where $f \in \text{Car}([0, T] \times \mathbb{R}^2)$, $J_i, M_i \in C(\mathbb{R})$.

A solution of problem (1)–(3) is a function $u \in APC^1$ which satisfies the impulse conditions (2), the periodic conditions (3) and for a.e. $t \in [0, T]$ fulfils equation (1).

Periodic BVP with fixed-time impulses

Basic assumptions: there exists a well-ordered pair of lower and upper functions σ_1, σ_2 of problem (1)–(3):

$$\sigma_1(t) \le \sigma_2(t), \quad t \in [0, T]. \tag{4}$$

$$\sigma_1''(t) \ge f(t, \sigma_1(t), \sigma_1'(t)), \tag{5}$$

$$\sigma_1(t_i+) = J_i(\sigma_1(t_i)), \quad \sigma_1'(t_i+) \ge M_i(\sigma_1'(t_i)), \quad i = 1, \dots, p, \tag{6}$$

$$\sigma_1(0) = \sigma_1(T), \quad \sigma_1'(0) \ge \sigma_1'(T), \tag{7}$$

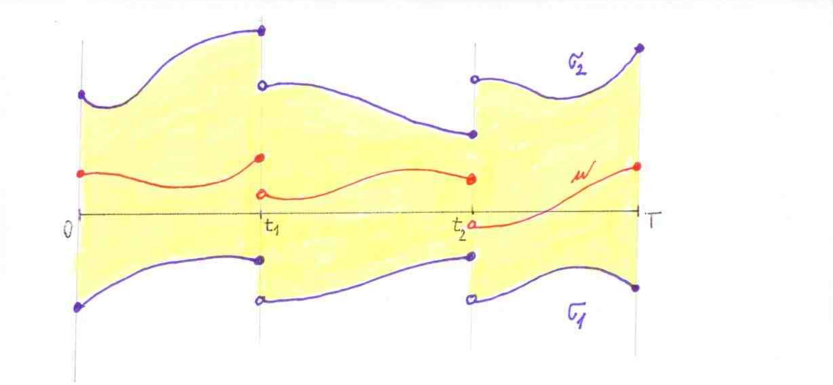
$$\sigma_2''(t) \le f(t, \sigma_2(t), \sigma_2'(t)), \tag{8}$$

$$\sigma_2(t_i+) = J_i(\sigma_2(t_i)), \quad \sigma_2'(t_i+) \le M_i(\sigma_2'(t_i)), \quad i = 1, \dots, p, \tag{9}$$

$$\sigma_2(0) = \sigma_2(T), \quad \sigma_2'(0) \le \sigma_2'(T). \tag{10}$$

Periodic BVP with fixed-time impulses

- We proved the solvability of problem (1)–(3) without the requirement of the monotonicity of the impulse functions $J_i, M_i,\ i = 1, \dots, p$.
- Proofs are based on the topological degree method and on a priori estimates of solutions of an auxiliary Dirichlet problem.



2001-Conference Equadiff in Prague

2001-Conference in London, Canada

2001-Conference in London, Canada

2002-Conference in Patras

2002-Conference in Patras

2002-Conference in Patras

Common papers 2001–2002

- Construction of lower and upper functions and their application to regular and singular periodic boundary value problems, *Nonlinear Analysis, TMA* 47 (2001), 6, 3937-3948.

- Localization of nonsmooth lower and upper functions for periodic boundary value problems, *Math. Bohemica* 127 (2002), 531-545.

- Impulsive periodic boundary value problem and topological degree. *Functional Differential Equations* No.3-4, 9 (2002), 471-498.

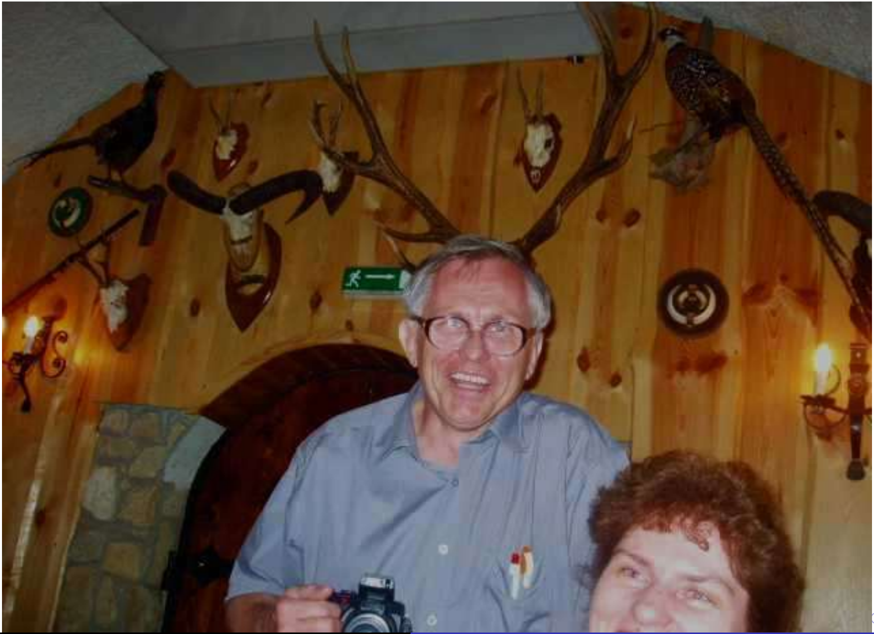
2003

Conferences in Miskolc, Szeged, Hasselt

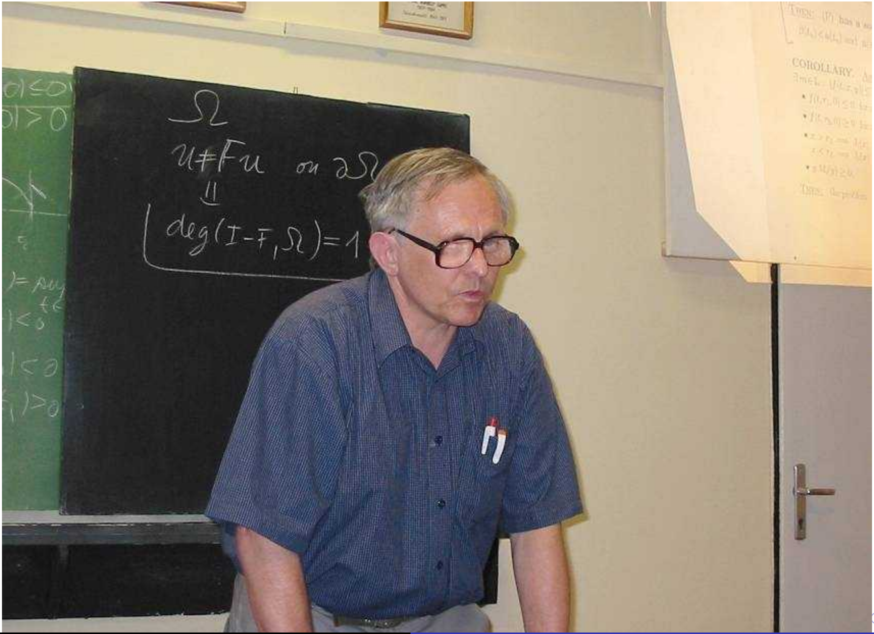
2003-Conference Miskolc

2003-Conference Miskolc

2003-Conference Miskolc



2003-Conference Miskolc



2003-Conference Szeged

2003-Conference Szeged

2003-Conference Szeged

2003-Conference Hasselt

2003-Conference Hasselt

Periodic BVP with fixed-time impulses

Basic assumptions: does not exist a well-ordered pair of lower and upper functions σ_1, σ_2 of problem (1)–(3) but there exists a not well-ordered pair of σ_1, σ_2:

$$\sigma_1(\tau) > \sigma_2(\tau), \quad \text{for some } \tau \in [0, T].$$

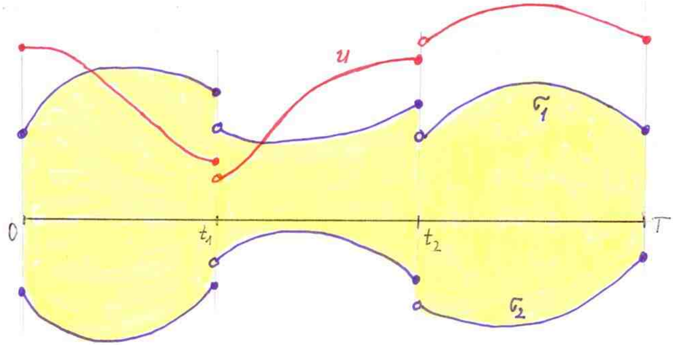
$$\sigma_1''(t) \geq f(t, \sigma_1(t), \sigma_1'(t)),$$
$$\sigma_1(t_i+) = J_i(\sigma_1(t_i)), \quad \sigma_1'(t_i+) \geq M_i(\sigma_1'(t_i)), \quad i = 1, \dots, p,$$
$$\sigma_1(0) = \sigma_1(T), \quad \sigma_1'(0) \geq \sigma_1'(T),$$

$$\sigma_2''(t) \leq f(t, \sigma_2(t), \sigma_2'(t)),$$
$$\sigma_2(t_i+) = J_i(\sigma_2(t_i)), \quad \sigma_2'(t_i+) \leq M_i(\sigma_2'(t_i)), \quad i = 1, \dots, p,$$
$$\sigma_2(0) = \sigma_2(T), \quad \sigma_2'(0) \leq \sigma_2'(T).$$

$$u''(t) = f(t, u(t), u'(t)),$$
$$u(t_i+) = J_i(u(t_i)), \quad u'(t_i+) = M_i(u'(t_i)), \quad i = 1, \dots, p,$$
$$u(0) = u(T), \quad u'(0) = u'(T),$$

where $f \in \text{Car}([0, T] \times \mathbb{R}^2)$, $J_i, M_i \in C(\mathbb{R})$.

- Well-ordered lower and upper functions to some new problem.
- Aditivity property of the Leray-Schauder topological degree.



2004

Conferences in Cluj, Orlando

2004-Conference Cluj

2004-Conference Cluj

2004-Conference Orlando

2004-Conference Orlando

Common papers 2003–2004

- Resonance and multiplicity in periodic boundary value problems with singularity. *Mathematica Bohemica* 128 (2003), 45-70.

- Existence results for impulsive second order periodic problems. *Nonlinear Analysis, TMA* 59 (2004), 133-146.

- Nonmonotone impulse effects in second order periodic boundary value problems. *Abstr. Appl. Anal.* 7 (2004), 577-590.

- Construction of non-constant lower and upper functions for impulsive second order periodic problems. *Electronic J. Qualit. Th. Diff. Equations* 19 (2004), 1-8.

2005-2006

Conferences in Gdansk, Iceland, Sarospatak

2005-Conference Gdansk

We investigated the solvability of the nonlinear periodic boundary value problem with ϕ-Laplacian and fixed-time impulses

$$(\phi(u'(t))') = f(t, u(t), u'(t)), \quad \text{a.e.} t \in [0, T], \tag{11}$$

$$u(t_i+) = J_i(u(t_i)), \quad u'(t_i+) = M_i(u'(t_i)), \quad i = 1, \ldots, p, \tag{12}$$

$$u(0) = u(T), \quad u'(0) = u'(T), \tag{13}$$

where $f \in \text{Car}([0, T] \times \mathbb{R}^2)$, $J_i, M_i \in C(\mathbb{R})$, ϕ is an increasing homeomorhpism, $\phi(\mathbb{R}) = \mathbb{R}$, $\phi(0) = 0$.

A solution of problem (11)–(13) is a function $u \in PC^1$ such that $\phi(u') \in APC$ and satisfies the impulse conditions (12), the periodic conditions (13) and for a.e. $t \in [0, T]$ fulfils equation (11).

Basic assumptions: there exists a well-ordered pair of lower and upper functions σ_1, σ_2 of problem (11)–(13):

$$\sigma_1(t) \le \sigma_2(t), \quad t \in [0, T]. \tag{14}$$

$$(\phi(\sigma_1'(t))' \ge f(t, \sigma_1(t), \sigma_1'(t)), \tag{15}$$

$$\sigma_1(t_i+) = J_i(\sigma_1(t_i)), \quad \sigma_1'(t_i+) \ge M_i(\sigma_1'(t_i)), \quad i = 1, \dots, p, \tag{16}$$

$$\sigma_1(0) = \sigma_1(T), \quad \sigma_1'(0) \ge \sigma_1'(T), \tag{17}$$

$$(\phi(\sigma_2'(t))' \le f(t, \sigma_2(t), \sigma_2'(t)), \tag{18}$$

$$\sigma_2(t_i+) = J_i(\sigma_2(t_i)), \quad \sigma_2'(t_i+) \le M_i(\sigma_2'(t_i)), \quad i = 1, \dots, p, \tag{19}$$

$$\sigma_2(0) = \sigma_2(T), \quad \sigma_2'(0) \le \sigma_2'(T). \tag{20}$$

Main ideas of the proof of the solvability of problem (11)–(13).

- We construct an auxiliary fixed point problem $\tilde{\mathcal{F}}u = u$ and prove $\deg(I - \tilde{\mathcal{F}}, \Omega_0) = 1$, where $\Omega_0 \subset PC^1$ is sufficiently large.
- Using the method of a priori estimates we construct a set $\Omega \subset \Omega_0$ and an operator $\mathcal{F}$ such that $\mathcal{F} = \tilde{\mathcal{F}}$ on $\overline{\Omega}$.
- By the excision property of the degree we prove $\deg(I - \tilde{\mathcal{F}}, \Omega) = 1$. Consequently, $\deg(I - \mathcal{F}, \Omega) = 1$.
- Having a fixed point $u \in \Omega$ of the operator $\mathcal{F}$, we prove that u is a solution of problem (11)–(13).

$$(\mathcal{F}x)(t) = \int_0^t \phi^{-1}\left(\mathcal{A}(\mathcal{N}x, (\mathcal{J}x)(T)) + (\mathcal{N}x)(s)\right) \mathrm{d}s$$
$$+x(0) + x'(0) - x'(T) + (\mathcal{J}x)(t),$$

where

$$(\mathcal{N}x)(t) = \int_0^t f(s, x(s), x'(s))\,\mathrm{d}s$$
$$+ \sum_{i=1}^{p} \left(\phi(M_i(x'(t_i))) - \phi(x'(t_i))\right) \chi_{(t_i,T]}(t),$$
$$(\mathcal{J}x)(t) = \sum_{i=1}^{p} (J_i(x(t_i)) - x(t_i))\, \chi_{(t_i,T]}(t),$$

$\mathcal{A} : PC \times \mathbb{R} \to \mathbb{R}$ is derived from ϕ.

2006-Conference Iceland

2006-Conference Iceland

2006-Conference Iceland

Periodic BVP with ϕ-Laplacian and fixed-time impulses

Basic assumptions: does not exist a well-ordered pair of lower and upper functions σ_1, σ_2 of problem (1)–(3) but there exists a not well-ordered pair of σ_1, σ_2:

$$\sigma_1(\tau) > \sigma_2(\tau), \quad \text{for some } \tau \in [0, T].$$

$$(\phi(\sigma_1'(t))' \geq f(t, \sigma_1(t), \sigma_1'(t)),$$
$$\sigma_1(t_i+) = J_i(\sigma_1(t_i)), \quad \sigma_1'(t_i+) \geq M_i(\sigma_1'(t_i)), \quad i = 1, \dots, p,$$
$$\sigma_1(0) = \sigma_1(T), \quad \sigma_1'(0) \geq \sigma_1'(T),$$

$$(\phi(\sigma_2'(t))' \leq f(t, \sigma_2(t), \sigma_2'(t)),$$
$$\sigma_2(t_i+) = J_i(\sigma_2(t_i)), \quad \sigma_2'(t_i+) \leq M_i(\sigma_2'(t_i)), \quad i = 1, \dots, p,$$
$$\sigma_2(0) = \sigma_2(T), \quad \sigma_2'(0) \leq \sigma_2'(T).$$

Not well-ordered pair of lower and upper functions

We proved the solvability of the problem

$$(11) \qquad (\phi(u'(t))' = f(t, u(t), u'(t)),$$

$$(12) \qquad u(t_i+) = J_i(u(t_i)), \quad u'(t_i+) = M_i(u'(t_i)), \quad i = 1, \dots, p,$$

$$(13) \qquad u(0) = u(T), \quad u'(0) = u'(T),$$

where $f \in \text{Car}([0, T] \times \mathbb{R}^2)$ is bounded by an integrable function, $J_i, M_i \in C(\mathbb{R})$ fulfil for $i = 1, \dots, p$

$$x > \sigma_1(t_i) \Rightarrow J_i(x) > J_i(\sigma_1(t_i)),$$

$$x < \sigma_2(t_i) \Rightarrow J_i(x) < J_i(\sigma_2(t_i)),$$

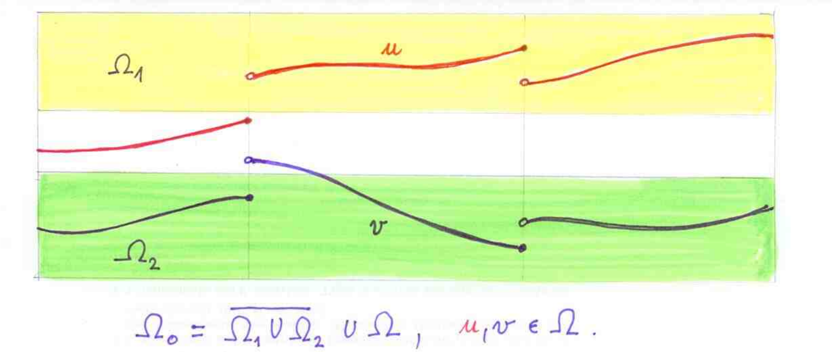
$$y \le \sigma_1'(t_i) \Rightarrow M_i(y) \le M_i(\sigma_1'(t_i)),$$

$$y \ge \sigma_2'(t_i) \Rightarrow M_i(y) \ge M_i(\sigma_2'(t_i)).$$

Not well-ordered pair of lower and upper functions

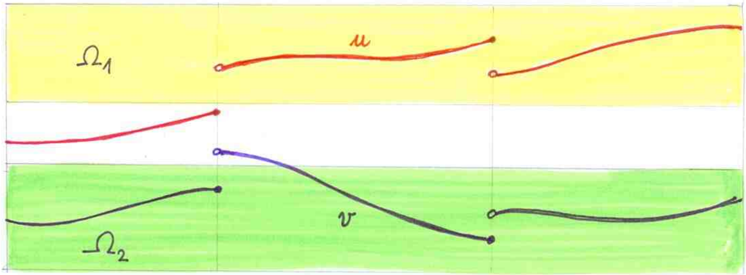
Main ideas of the proof of the solvability of problem (11)–(13)

- Construct an auxiliary operator $\tilde{\mathcal{F}}$ and introduce sets $\Omega_1, \Omega_2 \subset \Omega_0 \subset PC^1$, $\Omega = \Omega_0 \setminus \{\overline{\Omega_1 \cup \Omega_2}\}$.
- $\deg(I - \tilde{\mathcal{F}}, \Omega_0) = 1$, $\deg(I - \tilde{\mathcal{F}}, \Omega_1) = 1$, $\deg(I - \tilde{\mathcal{F}}, \Omega_2) = 1$.



Not well-ordered pair of lower and upper functions

- $\deg(I - \tilde{\mathcal{F}}, \Omega_1) + \deg(I - \tilde{\mathcal{F}}, \Omega_2) + \deg(I - \tilde{\mathcal{F}}, \Omega)$
  $= \deg(I - \tilde{\mathcal{F}}, \Omega_0).$
- $\deg(I - \tilde{\mathcal{F}}, \Omega) = -1 \neq 0$. Consequently $\tilde{\mathcal{F}}$ has a fixed point u in Ω and u is a solution of the corresponding auxiliary problem.
- Prove that u is a solution of the original problem (11)–(13).



$$\Omega_0 = \overline{\Omega_1 \cup \Omega_2} \cup \Omega, \quad u, v \in \Omega.$$

2006-Conference Sarospatak

Common papers 2005–2006

- Second order periodic problems with phi-Laplacian and impulses. *Nonlinear Analysis, TMA* 63 (2005), e257-e266.

- Periodic problems with phi-Laplacian involving non-ordered lower and upper functions. *Fixed Point Theory* 6 (2005), 99-112.

- Non-ordered lower and upper functions in second order impulsive periodic problems. *Dynamics of Continuous, Discrete and Impulsive Systems*, Ser. A, Math. Anal. 12 (2005), 397-415.

- Periodic singular problem with quasilinear differential operator. *Mathematica Bohemica* 131 (2006), 321-336.

Common books

Common books

- I. Rachůnková, S. Staněk, M. Tvrdý: Singularities and Laplacians in Boundary Value Problems for Nonlinear Ordinary Differential Equations. Handbook of Differential Equations. Ordinary Differential Equations, vol.3, pp. 607-723. Ed. by A. Caňada, P. Drábek, A. Fonda. Elsevier 2006.

- I. Rachůnková, S. Staněk, M. Tvrdý: Solvability of Nonlinear Singular Problems for Ordinary Differential Equations. Hindawi Publishing Corporation, New York, USA, 2009, 268 pages.

Some new results

State-dependent impulses

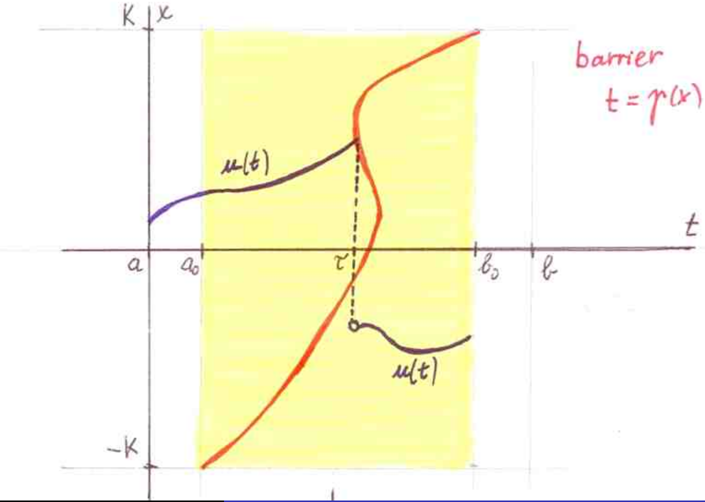
Common research with Jan Tomeček

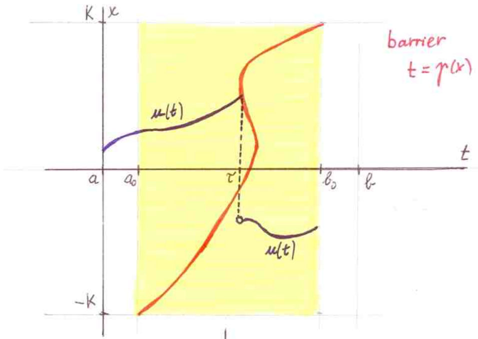
2008-Santiago de Compostela

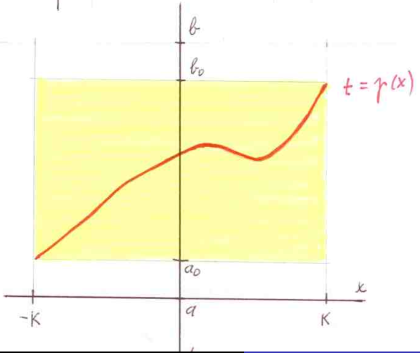
2011-Santiago de Compostela

State-dependent impulse of a left-continuous function $u(t)$

- a point τ of an impulse is given by $\tau = \gamma(u(\tau))$
- a graph of a barrier function $t = \gamma(x)$
- an impulse condition $u(\tau+) - u(\tau) = J(\tau, u(\tau))$



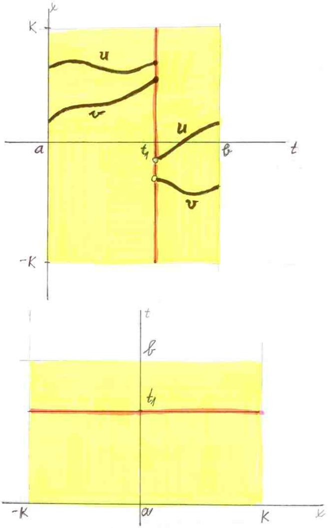

K
x
barrier
t = γ(x)
u(t)
t
a
a0
τ
b0
b
u(t)
-K


b
b0
t = γ(x)
a0
x
-K
a
K

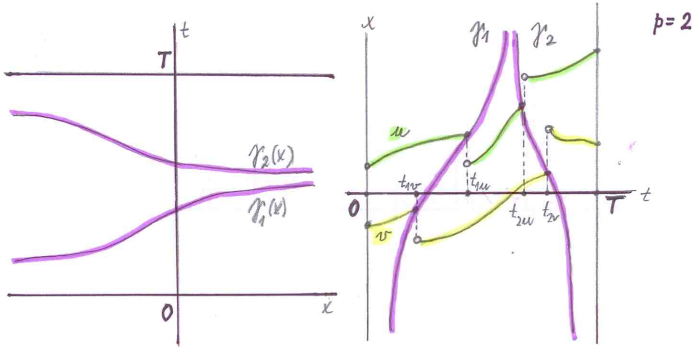
A special case - impulses at fixed times

A lot of papers for BVPs with impulses at fixed times.

A barrier is defined by a constant function $\gamma(x) \equiv t_1$.



- Barriers γ_1 and γ_2 are ordered.
- Different functions u, v have impulses at different points.



Formulation of problem - state-dependent impulses

$$a < \gamma_1(x) < \gamma_2(x) < \cdots < \gamma_p(x) < b,$$

$x \in D \subset \mathbb{R}^n$, $p \in \mathbb{N}$, $\quad \gamma_i \in \mathbb{C}(D; \mathbb{R})$, $i = 1, \dots, p$.

$$\begin{aligned}
&(1) && z'(t) = A(t)z(t) + f(t, z(t)), \text{ a.e. } t \in [a, b], \\
&(2_{SD}) && z(\tau_i+) - z(\tau_i) = J_i(z(\tau_i)), \text{ for } \tau_i = \gamma_i(z(\tau_i)), \\
& && \qquad\qquad i = 1, \dots, p, \\
&(3) && \ell(z) = c_0, \quad c_0 \in \mathbb{R}^n.
\end{aligned}$$

We assume that

$A \in \mathbb{L}^1([a, b]; \mathbb{R}^{n\times n})$, $f \in Car([a, b] \times \mathbb{R}^n; \mathbb{R}^n)$, $\quad J_i \in \mathbb{C}(\mathbb{R}^n; \mathbb{R}^n)$,

$i = 1, \dots, p$, $n \in \mathbb{N}$,

$\ell : \mathbb{G}_L([a, b]; \mathbb{R}^n) \to \mathbb{R}^n$ is linear bounded.

State-dependent impulses

- $\ell : \mathbb{G}_L([a,b];\mathbb{R}^n) \to \mathbb{R}^n$ is linear bounded.
- $\mathbb{G}_L([a,b];\mathbb{R}^n)$ is a Banach space of left-continuous regulated mappings.
- A mapping $u : [a,b] \to \mathbb{R}^n$ is left-continuous regulated on $[a,b]$ if for each $t \in (a,b]$ and each $s \in [a,b)$

$$u(t) = u(t-) = \lim_{\tau \to t-} u(\tau) \in \mathbb{R}^n, \quad u(s+) = \lim_{\tau \to s+} u(\tau) \in \mathbb{R}^n.$$

State-dependent impulses

Definition

$z : [a, b] \to \mathbb{R}^n$ is a solution of problem (1),(2_{SD}),(3), if

- for each $i \in \{1, \dots, p\}$ there exists a unique $\tau_i \in (a, b)$ such that $\gamma_i(z(\tau_i)) = \tau_i$,
- $a = \tau_0 < \tau_1 < \cdots < \tau_p < \tau_{p+1} = b$,
- the restrictions $z|_{[\tau_0,\tau_1]}$ and $z|_{(\tau_i,\tau_{i+1}]}$, $i = 1, \dots, p$, are absolutely continuous,
- z satisfies equation (1) for a.e. $t \in [a, b]$,
- z fulfils conditions (2_{SD}), (3).

By Definition, each solution z belongs to $\mathbb{G}_L([a, b]; \mathbb{R}^n)$.

- **M.Tvrdý**: Linear integral equations in the space of regulated functions, *Mathematica Bohemica* 123 (1998), 177–212.

$\ell : \mathbb{G}_L([a,b];\mathbb{R}^n) \to \mathbb{R}^n$ is a linear bounded operator if and only if there exist $K \in \mathbb{R}^{n\times n}$ and $V \in \mathbb{BV}([a,b];\mathbb{R}^{n\times n})$ such that

$$(4) \qquad \ell(z) = Kz(a) + \int_a^b V(t)\,\mathrm{d}[z(t)], \quad z \in \mathbb{G}_L([a,b];\mathbb{R}^n),$$

where the integral in (4) is the Kurzweil-Stieltjes integral.

Non-resonant case

Consider the corresponding linear homogeneous problem

$$z'(t) = A(t)z(t), \quad \ell(z) = 0. \tag{5}$$

Denote

$$\ell(Y) = (\ell(y_1), \dots, \ell(y_n))$$

and assume that

$$\det \ell(Y) \neq 0, \tag{6}$$

where Y is a fundamental matrix of the equation in (5).

Then there exists the Green's matrix G of the problem

(5) $$z'(t) = A(t)z(t), \quad \ell(z) = 0,$$

where

(4) $$\ell(z) = Kz(a) + \int_a^b V(t)\,\mathrm{d}[z(t)], \quad z \in \mathbb{G}_L([a,b];\mathbb{R}^n).$$

The matrix G has the form

$$G(t,\tau) = Y(t)H(\tau) + \chi_{(\tau,b]}(t)Y(t)Y^{-1}(\tau), \quad t,\tau \in [a,b],$$

where H is defined for $\tau \in [a,b]$ by

$$H(\tau) = -\left[\ell(Y)\right]^{-1}\left(\int_\tau^b V(s)A(s)Y(s)\,\mathrm{d}s \cdot Y^{-1}(\tau) + V(\tau)\right),$$

and

$$\ell(Y) = KY(a) + \int_a^b V(t)A(t)Y(t)\,\mathrm{d}t.$$

$$X=\left(\mathbb{W}^{1,\infty}([a,b];\mathbb{R}^n)\right)^{p+1}, \quad \mathcal{F}:\overline{\Omega}\subset X\to\left(\mathbb{C}([a,b];\mathbb{R}^n)\right)^{p+1},$$

$$\begin{aligned}(\mathcal{F}u)_k(t) = \int_a^b G(t,s)\sum_{i=1}^{p+1}\chi_{(\tau_{i-1},\tau_i)}(s)f(s,u_i(s))\,\mathrm{d}s \\ +\sum_{i=k}^{p} G_1(t,\tau_i)J_i(\tau_i,u_i(\tau_i)) \\ +\sum_{i=1}^{k-1} G_2(t,\tau_i)J_i(\tau_i,u_i(\tau_i)) \\ +Y(t)\,[\ell(Y)]^{-1}\,c_0, \quad k=1,\dots,p+1,\end{aligned}$$

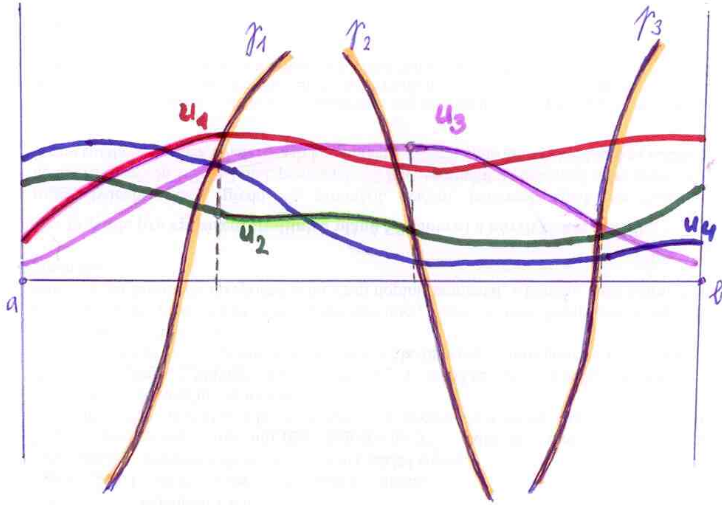
$$G_1(t,\tau_i)=Y(t)H(\tau_i), \quad G_2(t,\tau_i)=Y(t)\left(H(\tau_i)+Y^{-1}(\tau_i)\right),$$

$$\tau_i=\mathcal{P}_i u_i \quad \text{for } i=1,\dots,p, \quad \tau_0=a, \quad \tau_{p+1}=b.$$

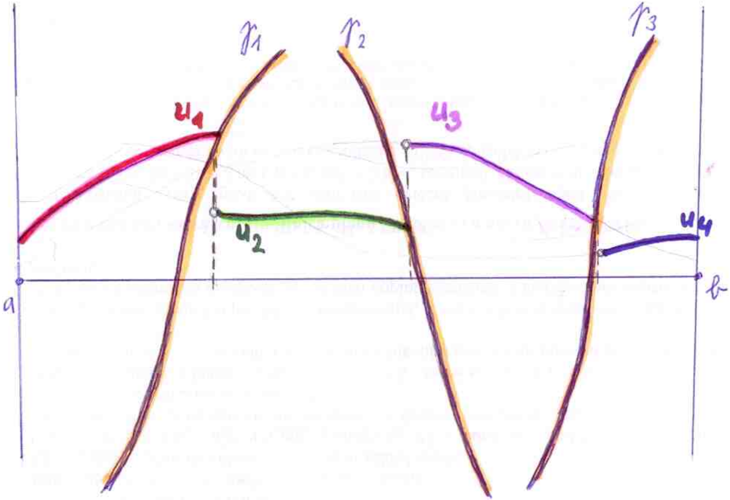
- Assume in addition that f is essentially bounded. Then the operator $\mathcal{F}$ maps $\overline{\Omega}$ to X. Unfortunatelly, $\mathcal{F}$ is not compact on $\overline{\Omega}$.
- Therefore we define an operator $\mathcal{G} : \overline{\Omega} \to X$ by

$$(\mathcal{G}u)_k(t) = \begin{cases} (\mathcal{F}u)_k(\tau_{k-1}) + \int_{\tau_{k-1}}^t f(s, u_k(s))\,\mathrm{d}s & \text{for } t < \tau_{k-1}, \\ (\mathcal{F}u)_k(t) & \text{for } \tau_{k-1} \le t \le \tau_k, \\ (\mathcal{F}u)_k(\tau_k) + \int_{\tau_k}^t f(s, u_k(s))\,\mathrm{d}s & \text{for } t > \tau_k, \end{cases}$$

  where $t \in [a, b]$, $k = 1, \dots, p+1$. Then $\mathcal{G}$ is compact on $\overline{\Omega}$.
- If all data functions are sufficiently small and derivatives of barrier functions are sufficiently small, then $\mathcal{G}$ has a fixed point in Ω.
- A solution of the original problem (1), (2_{SD}), (3) can be constructed from this fixed point.



γ_1
γ_2
γ_3
u_1
u_2
u_3
u_4
a
b



γ_1
γ_2
γ_3
u_1
u_2
u_3
u_4
a
b

Thank you for your attention!