# On the existence of positive periodic solutions to a singular problem modeling valveless pumping with friction and its asymptotic stability

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The talk is based on the joint paper [4] with José Ángel Cid and Georg Propst recently published in Physica D.

We consider the periodic problem

$$u'' + a\,u' = \frac{1}{u}\left(e(t) - b\,(u')^2\right) + c, \quad u(0) = u(T),\ u'(0) = u'(T)\,, \tag{P}$$

where $T, a, c > 0$, $b > 1$ and the forcing term $e$ is essentially bounded on $[0, T]$. As shown by G. Propst [4], this problem describes valveless pumping in the one pipe - one tank system. In general, valveless pumping assists in fluid transport in various biomedical and engineering systems and it describes e.g. blood circulation in the cardiovascular system when the heart's valves fail or when the embryonic vertebrate heart begins pumping blood long before the development of discernable chambers and valves. Despite the formal simplicity of the differential equation in (P), the singularity on the right-hand side makes it more difficult to analyze and, in fact, there is a lack of general existence results for (P). From the physical point of view we are interested in the search of positive solutions.

It is easy to see that (P) has a positive solution $u(t) > 0$ only if the mean value $\bar{e}$ of $e$ is positive. The aim of the contribution is to deliver conditions sufficient for the existence and asymptotic stability of a positive solution to (P).

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