Generalized elementary functions

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Given a bounded variation function $P:\mathbb{R}\to\mathbb{C}$, we introduce the generalized exponential function $e_{\mathrm{d}P}(\cdot,t_0)$ as the solution of the linear integral equation

$$z(t)=1+\int_{t_0}^{t} z(s)\,\mathrm{d}P(s),\quad t\in\mathbb{R},$$

where the integral on the right-hand side is the Kurzweil-Stieltjes integral.

We also define the generalized hyperbolic and trigonometric functions as follows:

$$\cosh_{\mathrm{d}P}(t,t_0)=\frac{e_{\mathrm{d}P}(t,t_0)+e_{\mathrm{d}(-P)}(t,t_0)}{2},\quad \sinh_{\mathrm{d}P}(t,t_0)=\frac{e_{\mathrm{d}P}(t,t_0)-e_{\mathrm{d}(-P)}(t,t_0)}{2},$$

$$\cos_{\mathrm{d}P}(t,t_0)=\frac{e_{\mathrm{d}(iP)}(t,t_0)+e_{\mathrm{d}(-iP)}(t,t_0)}{2},\quad \sin_{\mathrm{d}P}(t,t_0)=\frac{e_{\mathrm{d}(iP)}(t,t_0)-e_{\mathrm{d}(-iP)}(t,t_0)}{2i}.$$

We present some basic properties of these generalized functions, and show that elementary functions on time scales represent a special case of our definitions. In fact, we are able to extend the usual definitions of the time scale elementary functions from rd-continuous to Lebesgue Δ-integrable arguments.

This is a joint work with Giselle Antunes Monteiro (Mathematical Institute, Academy of Sciences of the Czech Republic).

Reference

[1] G. Antunes Monteiro, A. Slavík: *Generalized elementary functions.* Journal of Mathematical Analysis and Applications 411 (2014), 838–852.