Nonexistence of blowing-up and exponential stability of solutions to fractional perturbations of delay differential equations

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We present some results on the non-existence of blowing-up solutions and on the exponential stability of the zero solution to fractional perturbations of ordinary and delay differential equations. First we consider the initial value problem

$$\dot{x}(t) = Ax(t) + f(t, x(t), x(t-T), I^{\alpha_1}[g_1 x](t), \ldots, I^{\alpha_m}[g_m x](t)),\ t > 0,$$

$$x(t) = \Phi(t),\ t \in [-T, 0],\ T > 0,$$

where $\Phi \in C_T := C([-T, 0], X), x \in X, X$ is a Banach space, A is the infinitesimal generator of a strongly continuous semigroup on X, f, g are continuous mappings,

$$I^{\alpha_i}[g_i x](t) := \frac{1}{\Gamma(\alpha_i)} \int_0^t (t-s)^{\alpha_i - 1} g_i(s, x(s)) ds,\ 0 < \alpha_i < 1,\ i \in \{1, 2, \ldots, m\}$$

is the Riemann-Liouville integral of the function $[g_i x](t) := g_i(t, x(t))$.Then we will discuss these problems also for the finite dimensional system

$$\dot{x}(t) = Ax(t) + Bx(t-T) +$$

$$f(t, x(t), x(t-T), I^{\alpha_1}[g_1](t), \ldots, I^{\alpha_m}[g_m x](t)),\ t > 0, x \in \mathbb{R}^n,$$

$$x(t) = \Phi(t),\ t \in [-T, 0],\ T > 0,$$

where A, B are permutable matrices. The non-fractional case when the nonlinearity f is independent of the Riemann-Liouville integrals is studied in the paper [1].

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References

[1] M. Medveď, M. Pospísil, L. Škripková , *Stability and the nonexistence of blowing-up solutions of nonlinear delay systems with linear parts defined by permutable matrices, Nonlinear Analysis:Theory, Methods and Applications* , **74** (2011), 3903-3911.