Boundary value problems for differential systems coming from models of burglary of houses

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Using Leray-Schauder degree arguments, existence results for positive solutions (A, N) are proved for ordinary differential systems of the form

$$\eta(x)[A - A^0(x)]'' - A + A^0(x) + Nf(x, A) = 0,$$

$$N'' + [g(x, A, A')N]' - \omega^2(N - 1) = 0,$$

and of the form

$$\eta[A - A^0(x)]'' - A + A^0(x) + NA = 0,$$

$$\left(N' - 2N\frac{A'}{A}\right)' - NA + A^1(x) - A^0(x) = 0,$$

with either Neumann or periodic boundary conditions. Here A^0 is of class C^2 and η, A^1, f, g continuous functions satisfying some further mild conditions.

The form of the systems and the assumptions upon η, A_0, f and g are motivated by some mathematical models for the burglary of houses. The requested a priori estimates are obtained by some unusual combination of pointwise and L^1-estimates.

Similar results holds for the radial solutions on an annulus Ω for systems of partial differential equations of the form

$$\eta(x)\Delta[A - A^0(x)] - A + A^0(x) + Nf(x, A) = 0 \text{ in } \Omega$$

$$\Delta N + \nabla \cdot [N\nabla h(A)] - \omega^2(N - 1) = 0 \text{ in } \Omega, \quad \frac{\partial A}{\partial \nu} = \frac{\partial N}{\partial \nu} = 0 \text{ on } \partial\Omega.$$

This is a joint work with M. Garcia-Huidobro and R. Manásevich.

References

[1] *M. Garcia-Huidobro, R. Manásevich and J. Mawhin* : Existence of solutions for a 1-D boundary value problem coming from a model for burglary, Nonlinear Anal. RWA *14* (2013), 1939-1946

[2] *M. Garcia-Huidobro, R. Manásevich and J. Mawhin* : Existence of solutions for a nonlinear boundary value problem arising from a model for burglary, Appl. Math. Letter, published on-line

[3] *M. Garcia-Huidobro, R. Manásevich and J. Mawhin* : Radial solutions of a Neumann problem coming from a burglary model, in preparation