Half-linear Euler type differential equation with periodic coefficients

The critical half-linear Euler differential equation is an equation

$$\text{(E)} \qquad \big(\Phi(x')\big)' + \frac{\gamma_p}{t^p}\Phi(x) = 0, \quad \Phi(x) = |x|^{p-2}x,\ p > 1,$$

where $\gamma_p = \left(\frac{p-1}{p}\right)^p$. Euler half-linear differential equation is a typical example of the so-called *conditionally oscillatory* equation. If we replace the constant γ_p in this equation by a real parameter γ, then the equation is oscillatory if and only if $\gamma > \gamma_p$.

We will study the influence of various perurbations of (E) on oscillatory behavior of the perturbed equation. Typically, we consider the equation

$$\text{(P)} \quad \left(\left(1+\sum_{j=1}^{n}\frac{\alpha_j(t)}{\mathrm{Log}_k^2(t)}\right)\Phi(x')\right)' + \frac{1}{t^p}\left(\gamma_p+\sum_{j=1}^{n}\frac{\beta_j(t)}{\mathrm{Log}_j^2(t)}\right)\Phi(x) = 0$$

with T-periodic functions α_j, β_j and

$$\mathrm{Log}_j(t) = \prod_{i=1}^{j}\log_i(t), \quad \log_i(t) = \log_{i-1}(\log t), \quad \log_1(t) = \log t.$$

We establish an exact formula involving the mean values of α_j, β_j over the period

$$\bar{\alpha}_j = \frac{1}{T}\int_0^T \alpha_j(t)\,dt, \quad \bar{\beta}_j = \frac{1}{T}\int_0^T \beta_j(t)\,dt,$$

which implies (non)oscillation of (P).