Nonoscillation and stability of delay differential equations without damping term

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Delays, arising in nonoscillatory and stable ordinary differential equations, can induce oscillation and instability of their solutions. That is why the traditional direction in the study of nonoscillation and stability of delay equations is to establish a smallness of delay, allowing delay differential equations to preserve these convenient properties of ordinary differential equations with the same coefficients. In this talk, we find cases in which delays, arising in oscillatory and asymptotically unstable ordinary differential equations, induce nonoscillation and stability of delay equations. We demonstrate that, although the ordinary differential equation

$$x''(t) + c(t)x(t) = 0,$$

can be oscillatiting and asymptoticaly unstable, the delay equation

$$x''(t) + a(t)x(t - h(t)) - b(t)x(t - g(t)) = 0, \quad \text{where } c(t) = a(t) - b(t),$$

can be nonoscillating and exponentially stable. Results on nonoscillation and exponential stability of delay differential equations are obtained.