Equations with involutions

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This talk is devoted to the study of the following first order functional equation, coupled with periodic boundary value conditions:

$$x'(t) = f(t, x(t), x(\varphi(t))), \text{ for a. e. } t \in J, \qquad x(\inf J) = x(\sup J).$$

Here function $\varphi : J \to J$ is such that $\varphi \circ \varphi = \mathrm{Id}$, and it is called an involution.

To this end, we consider the linear equation

$$x'(t) + a(t)\, x(t) + b(t)\, x(\varphi(t)) = h(t), \text{ for a. e. } t \in J, \qquad x(\inf J) = x(\sup J).$$

After constructing an equivalence between all the involution equations, we concentrate our study in the particular case of the reflection operator $\varphi(t)) = -t$ and $J = [-T, T]$.

We obtain some estimations on the norm of a and b (optimal in some cases) to ensure that the linear problem has a unique solution and it has constant sign on $[-T, T]$. In this way, we automatically establish maximum and anti-maximum principles for the linear operator. In some particular situations we are able to obtain the exact expression of the Green's function.

The existence results for the nonlinear problem follow from iterative techniques and fixed point theorems in cones. In some of the given results, the Green's function is allowed to change sign on its square of definition.

References

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