On the BVPs for Φ-Laplacian type equation

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Consider the boundary value problems (BVPs) for Φ-Laplacian equation $d/dt(\Phi(t,x')) + f(t,x) = 0$ (i), where $t \in I := [a,b]$, functions $f \in C(I \times \mathbb{R}, \mathbb{R})$ and $\Phi \in C(I \times \mathbb{R}, \mathbb{R})$ satisfy the Lipschitz conditions with respect to x and x' respectively and Φ is monotone with respect to x', together with the boundary conditions $x(a)\cos\alpha - \Phi(a, x'(a))\sin\alpha = 0,\ x(b)\cos\beta - \Phi(b, x'(b))\sin\beta = 0$ (ii), where $0 \le \alpha < \pi,\ 0 < \beta \le \pi$.

We investigate existence and multiplicity of solutions to the BVP (i), (ii) reducing the given problem to a problem for a two-dimensional differential system $x' = \Phi^{-1}(t,y),\ y' = -f(t,x)$ (iii) with the boundary conditions $x(a)\cos\alpha - y(a)\sin\alpha = 0$, $x(b)\cos\beta - y(b)\sin\beta = 0$ (iv) and using the quasilinearization process described in [1], [2], [3].

We extract the linear parts (which are non-resonant with respect to the boundary conditions (iv))and represent the system (iii) in an equivalent quasi-linear form $x' - k^2 y = h(t,y)$, $y' + l^2 x = g(t,x)$ (v). We define a type of non-resonance of the extracted linear part with respect to the boundary conditions (iv) and define an oscillatory type of a solution to the BVP (v), (iv). The main result is the following.

Theorem. Quasi-linear problem (v), (iv) has a solution, the oscillatory type of which corresponds to the type of non-resonance of the extracted linear part.

We show that BVP (i), (ii) under certain conditions can be reduced to the quasi-linear problem of the type (v), (iv) multiply with essentially different linear parts, and, consequently, has multiple solutions of different types. (A type of a solution to the BVP (i), (ii) is induced by an oscillatory type of a solution to the respective quasi-linear problem (v), (iv).)

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References

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