On solvability of certain boundary value problems for second order linear functional differential equations

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Consider the equation

$$u''(t) = \ell(u)(t) + q(t), \tag{1}$$

where $\ell : C([a, b]; \mathbb{R}) \to L([a, b]; \mathbb{R})$ is a linear bounded operator and $q \in L([a, b]; \mathbb{R})$. Together with equation (1) consider one of the following boundary condition:

$$u(a) = c_1, \quad u(b) = c_2, \tag{2}$$

$$u(a) = c_1, \quad u'(b) = c_2, \tag{3}$$

$$u(a) = c_1, \quad u(b) = u(t_0) + c_2, \tag{4}$$

where $t_0 \in \,]a, b[\,$ and $c_1, c_2 \in \mathbb{R}$.

Efficient conditions for the solvability of problems (1), (i) $(i \in \{2, 3, 4\})$ are established.