Anti-maximum principle for p-Laplacians and its application to weakly singular periodic problems

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The following ANTIMAXIMUM PRINCIPLE is well known:

$$\left.\begin{array}{l} q\in L_1[0,T],\ 0\le q\le (\frac{\pi}{T})^2,\ \overline{q}>0, \\ v\in C^1[0,T], \quad v'\in AC[0,T], \\ v''+q\,v\ge 0 \text{ a.e. on } [0,T],\ v(0)=v(T),\ v'(0)\ge v'(T) \end{array}\right\} \Rightarrow v\ge 0 \ \text{ on } [0,T].$$

Its validity is related to the explicit knowledge of the corresponding Green's function

$$G(t,s)=\frac{T}{2\,\pi}\,\sin\Big(\frac{\pi}{T}\,|t-s|\Big) \quad \text{for} \ \ t,s\in[0,T]$$

for the case $q\equiv\left(\frac{\pi}{T}\right)^2$. ANTIMAXIMUM PRINCIPLE is helpful for getting a priori estimates from below of the solutions to singular nonlinear boundary value problems of the form

$$u''=f(t,u), \quad u(0)=u(T),\ u'(0)=u'(T),$$

where the right hand side $f(t,u)$ can have a weak singularity for $u=0$.

In our talk we give a sketch of the proof of ANTIMAXIMUM PRINCIPLE for the quasilinear problem

$$|v'|^{p-2}\,v'+q\,|v|^{p-2}\,v\ge 0 \ \text{ a.e. on } \ [0,T], \quad v(0)=v(T),\ v'(0)\ge v'(T).$$

(Of course, in this case, no tools like Green's function are available.)

In addition, we will present new existence results for the problem

$$\big(|u'|^{p-2}\,u'\big)'=g(u)+e(t), \quad u(0)=u(T), \quad u'(0)=u'(T),$$

where $1<p<\infty$, $e\in L_1[0,T]$ and $g\in C(0,\infty)$ can have a weak repulsive space singularity at $x=0$, i.e. $\limsup_{x\to 0+} g(x)=\infty$ and $\left|\lim_{x\to 0+}\int_x^1 g(\xi)\,\mathrm{d}\xi\right|<\infty$ may hold.

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A. Cabada, A. Lomtatidze and M. Tvrdý. Periodic problem with quasilinear differential operator and weak singularity. *Advanced Nonlinear Studies*, to appear.

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