Positive solutions of singular mixed boundary value problems with time and space singularities

Svatoslav Staněk

Olomouc, Czech Republic

We say that $h \in Car((0,T) \times \mathcal{D})$, $\mathcal{D} = (0,\infty) \times \mathbb{R}$, has a time singularity at $t = 0$ ($t = T$) if there exists $(x,y) \in \mathcal{D}$ such that

$$\int_0^{\nu} |h(t,x,y)| \, \mathrm{d}t = \infty \quad \Big(\int_{T-\nu}^{T} |h(t,x,y)| \, \mathrm{d}t = \infty \Big)$$

for all sufficiently small $\nu > 0$. If $\lim_{x \to 0^+} |h(t,x,y)| = \infty$ for a.e. $t \in [0,T]$ and some $y \in \mathbb{R}$ then we say that h has a space singularity at $x = 0$. We say that $x = 0$ is a weak (strong) space singularity of h if there exists $y \in \mathbb{R}$ such that for a.e. $t \in [0,T]$ and all sufficiently small $\rho > 0$ the relation

$$\int_0^{\rho} |h(t,x,y)| \, \mathrm{d}x < \infty \quad \Big(\int_0^{\rho} |h(t,x,y)| \, \mathrm{d}x = \infty \Big)$$

holds. The contribution deals with the singular mixed boundary value problem

$$u'' = p(u')[f(t,u,u') - r(t)], \tag{1}$$

$$u(0) = 0, \quad u'(T) = 0, \tag{2}$$

where $f \in Car((0,T) \times \mathcal{D})$ admits a time singularity at $t = 0$ and/or at $t = T$ and a strong space singularity at $u = 0$. We give conditions on the functions p, f and r in equation (1) which guarantee the existence of a positive (on $(0,T]$) solution $u \in AC^1[0,T]$ and the maximal solution $\overline{u} \in AC^1[0,T]$ of problem (1), (2). Here $\overline{u}$ is called the maximal positive solution of problem (1), (2) if $\overline{u} \geq u$ on $[0,T]$ for all its positive solutions u.