On existence and uniqueness of solution of optimal control problems for distributed systems unsolved with respect to the time derivative

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Let $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{U}$ be Hilbert spaces, operators $L \in \mathcal{L}(\mathcal{X};\mathcal{Y})$, $\ker L \neq \{0\}$, $M \in \mathcal{C}l(\mathcal{X};\mathcal{Y})$, $B \in \mathcal{L}(\mathcal{U};\mathcal{Y})$. Consider optimal control problem

$$N(x(0) - x_0) = 0, \tag{1}$$

$$L\dot{x}(t) = Mx(t) + y(t) + Bu(t), \tag{2}$$

$$u \in \mathfrak{U}_\partial, \tag{3}$$

$$J(x,u) = \frac{1}{2}\|x - w\|^2_{H^{r_1}(0,T;\mathcal{X})} + \frac{K}{2}\|u - u_0\|^2_{H^{r_2}(0,T;\mathcal{U})} \to \inf, \tag{4}$$

where $r_1 \in \{0,1\}$, $r_2 \in \{0,1,2,\dots\}$, $x_0 \in \mathcal{X}$ is a given vector, y, w, u_0 are given functions, u is control function, $K \geq 0$, $\mathfrak{U}_\partial$ is a nonempty convex closed subset of control functions space $H^{r_2}(0,T;\mathcal{U})$.

In the case when operator $N = I$, problem (1), (2) is the Cauchy problem. Besides, in applications systems often arise that described in initial moment by the general Showalter condition (1), when $N = P$ is projector along the kernel of resolving semigroup of homogeneous equation (2) on the phase space of the equation.

In the case $K = 0$ such problems often called as problems of hard control.

For research of problems of the form (1) – (4) scheme is applicated that allows to use only the property of nontriviality of considered system and properties of minimized functional for proof of existence and uniqueness of solution (see [1], [2]).

Abstract results applied to problems for some classes of partial derivative equations or systems of equations that unsolved with respect to the time derivative.

Consider the case of $P = I$, $K > 0$, $r_1 = 1$, $r_2 \in \{0,\dots,p+1\}$ with assumption of strong (L,p)-radiality of operator M that guarantees the existence of strongly continuous resolving semigroup of homogeneous equation (2). Denote for $x_0 \in \mathrm{dom}M$, $y \in H^{p+1}(0,T;\mathcal{Y})$ the set of control functions $u \in H^{p+1}(0,T;\mathcal{U})$ satisfying the condition $(I-P)x_0 = -\sum\limits_{k=0}^{p+1}(M_0^{-1}L_0)^k M_0^{-1}(I-Q)(Bu^{(k)}(0) + y^{(k)}(0))$ by $H_\partial(x_0,y)$, and $\mathcal{Z}_{r_2} \equiv \{z \in H^1(0,T;\mathcal{X}) : L\dot{z} - Mz \in H^{r_2}(0,T;\mathcal{Y})\}$. Following result is obtained.

Theorem 1. *Let operator M be strongly (L,p)-radial, $\mathfrak{U}_\partial \cap H_\partial(x_0,y) \neq \emptyset$. Then there exists a unique solution $(\hat{x},\hat{u}) \in \mathcal{Z}_{r_2} \times H^{r_2}(0,T;\mathcal{U})$ of the problem* (1) – (4).

References

[1] Plekhanova M.V., Fedorov V.E. An optimal control problem for a class of degenerate equations. J. of Computer and System Sciences International, 2004, v. 43, no. 5, p. 698-702.

[2] Plekhanova M.V., Fedorov V.E. An optimality criterion in a control problem for a Sobolev type equation. J. of Computer and System Sciences International, 2007, v. 46, no. 2, p. 248-254.