New solvability conditions for a nonlocal boundary value problem for nonlinear functional differential equations

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We consider boundary value problem

$$u'(t) = F(u)(t), \qquad u(a) = h(u) + \varphi(u),$$
(1)

where $F: C([a, b]; \mathbb{R}) \to L([a, b]; \mathbb{R})$ is a continuous operator satisfying the Carathéodory conditions, $h: C([a, b]; \mathbb{R}) \to \mathbb{R}$ is a linear bounded functional and $\varphi: C([a, b]; \mathbb{R}) \to \mathbb{R}$ is a continuous functional such that, for every r > 0, there exists $M_r \in \mathbb{R}_+$ satisfying

$$|\varphi(v)| \le M_r$$
 for $v \in C([a,b]; \mathbb{R}), ||v||_C \le r$.

We present new effective sufficient conditions for the existence of a solution of the boundary value problem (1).