One class of nonlinear equations with Sobolev type singularity

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Let $\mathcal{X}$ and $\mathcal{Y}$ be Banach spaces, operators $L \in \mathcal{L}(\mathcal{X};\mathcal{Y})$, $\ker L \neq \{0\}$, $M \in \mathcal{C}l(\mathcal{X};\mathcal{Y})$. Suppose that operator M is strongly (L,p)-sectorial [1], then $\mathfrak{U} = \mathfrak{U}^0 \oplus \mathfrak{U}^1$, $\mathfrak{F} = \mathfrak{F}^0 \oplus \mathfrak{F}^1$ and there exists sectorial operator $L_1^{-1}M_1 \in \mathcal{C}l(\mathcal{U}^1)$. Take sufficiently great $b \in \rho(L_1^{-1}M_1) \cap \mathbb{R}$, $\alpha \in [0,1)$, $\mathcal{U}^1_\alpha = \mathrm{dom}((bI - L_1^{-1}M_1)^\alpha)$ and denote by P the projector along $\mathcal{U}^0$ on $\mathcal{U}^1$. Let $N : J \times U \to \mathcal{Y}$ is nonlinear operator, $U = U^0 \oplus U^1$, $J \subset \mathbb{R}$ and $U^1 \subset \mathcal{U}^1_\alpha$ are opened sets, $U^0 \subset \mathcal{U}^0$. Consider the Cauchy problem $u(0) = u_0$ for the equation

$$L\dot{u}(t) = Mu(t) + N(t, u(t)). \tag{1}$$

Theorem 1. *Let operator M be strongly (L,p)-sectorial, mapping $N : J \times U \to \mathcal{F}$ satisfies to local Hölder condition with respect to t and satisfies to local Lipschitz condition with respect to $v = Pu$ on the set $J \times U \subset \mathbb{R} \times \mathcal{U}^0 \oplus \mathcal{U}^1_\alpha$, $\alpha \in [0,1)$, and $\mathrm{im}N \subset \mathfrak{F}^1$. Then for any $(t_0, u_0) \subset J \times U^1$ there exists $T = T(t_0, u_0)$, such that the Cauchy problem for the equation* (1) *has a unique solution on $(t_0, t_0 + T)$.*

Remark 1. In the papers of G.A.Sviridyuk and his coathors (for example, [2, 3]) such equations was considered in the case of stationary and smooth operator N.

Let $\Omega \subset \mathbb{R}^s$ is a bounded region with a boundary $\partial\Omega$ of the class C^∞, $\lambda, \alpha, \beta \in \mathbb{R}$. Denote $Aw = \Delta w$, $\mathrm{dom}A = H^2_{\frac{\partial}{\partial n}+\lambda}(\Omega) \subset L_2(\Omega)$ and assume that $-\beta \notin \sigma(A)$. Take $\mathcal{U} = \mathcal{F} = (L_2(\Omega))^2$, sufficiently great $b \in \rho(A) \cap \mathbb{R}$, $\alpha \in [0,1)$ and $\mathcal{U}^1_\alpha = \mathrm{dom}((bI - A)^\alpha)$. Applicating the Theorem 1 we can investigate the boundary value problem

$$\theta(x, t_0) + \varphi(x, t_0) = u_0(x) \in \mathcal{U}^1_\alpha, \quad x \in \Omega,$$

$$\frac{\partial\theta}{\partial n}(x,t) + \lambda\theta(x,t) = \frac{\partial\varphi}{\partial n}(x,t) + \lambda\varphi(x,t) = 0, \quad (x,t) \in \partial\Omega \times (t_0, T),$$

for the system of the type of phase field equations system

$$\theta_t(x,t) + \varphi_t(x,t) = \Delta\theta(x,t) + N(t, \theta, \varphi), \quad (x,t) \in \Omega \times (t_0, T),$$

$$\Delta\varphi(x,t) + \beta\varphi(x,t) + \alpha\theta(x,t) = 0, \quad (x,t) \in \Omega \times (t_0, T).$$

References

[1] Sviridyuk G.A., Fedorov V.E. On the identities of analytic semigroups of operators with kernels. Sib. Math. J., 1998, v. 39, no. 3, p. 522-533.

[2] Sviridyuk G.A., Sukacheva T.G. Phase spaces of a class of operator semilinear equations of Sobolev type. Differential Equations, 1990, v. 26, no. 2, p. 188-195.

[3] Sviridyuk G.A. Phase portraits of Sobolev type semilinear equations with a relatively strongly sectorial operator. St. Petersburg Math. J., 1995, v. 6, no. 5, p. 1109-1126.