On the solvability of boundary value problems for the nonlinear systems of generilized ordinary differential equations and for impulsive equations

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Let $\sigma_1, \dots, \sigma_n \in \{-1, 1\}$; $A : [-a, a] \to R^{n\times n}$ be a matrix-function with bounded total variation components, $f : [-a, a] \times R^n \to R^n$ be a vector-function belonging to the Carathéodory class corresponding to the matrix-function A, and $\varphi_i : \mathrm{BV}([-a, a], R^n) \to R$ $(i = 1, \dots, n)$ be continuous functionals.

For the system of nonlinear generalized ordinary differential equations

$$dx(t) = dA(t) \cdot f(t, x(t)), \tag{1}$$

where $x = (x_i)_{i=1}^n$, the multi-point boundary value problem

$$x_i(-\sigma_i a) = \varphi_i(x_1, \dots, x_n) \quad (i = 1, \dots, n) \tag{2}$$

is considered.

Necessary and sufficient as well as effective sufficient conditions are established for the solvability of the boundary value problem (1), (2).

The results are realized for the impulsive system with finite and fixed points of impulses actions

$$\frac{dx}{dt} = f(t, x(t)) \ \text{ for almost all } \ t \in [a, b] \setminus \{\tau_k\}_{k=1}^{m_0},$$
$$x(\tau_k+) - x(\tau_k-) = I_k(x(\tau_k-)) \ \ (k = 1, \dots, m_0),$$

with boundary conditions (2), where $f \in K([-a, a] \times R^n, R^n)$, $I_k : R^n \to R^n$ $(k = 1, \dots, n)$ are arbitrary operators, and $-a < \tau_1 < \dots < \tau_{m_0} \le a$.

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