On existence of solution to odd-order nonlinear Emden-Fowler type equation with given number of zeroes on prescribed interval.

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1 Introduction

The problem of existence of solutions with given number of zeros on prescribed domain to Emden–Fowler type equations is investigated.

Consider the equation

$$y^{(n)} = -p(t, y, y', ..., y^{(n-1)})|y|^k \operatorname{sgn} y, \tag{1}$$

where n is odd, $k \in (0,1) \cup (1, +\infty)$, function $p(t, y_1, y_2, y_3, ..., y_n)$ is continious and it is Lipschitz continuous in $(y_1, y_2, y_3, ..., y_n)$, and satisfy inequalities $0 < m \leqslant p(t, y_1, y_2, ..., y_n) \leqslant M < +\infty$.

In [1] equations of the third– and the fourth– order with constant coefficient and $k \in (0,1) \cup (1, +\infty)$ was investigated, using [2]. By using methods [3] higher order differential equation (1) with constant potential and regular nonlinearity $k > 1$ is investigated in [4]. The third order differential equation (1) with non-constant potential and regular nonlinearity is considered in [5].

Now we generalize obtained results to odd order differential equation (1) with regular and singular nonlinearity.

2 Main results

Theorem 2.1 *For any odd n, real $k \in (0,1) \cup (1, +\infty)$, real m, M: $0 < m < M < +\infty$, real a, b: $-\infty < a < b < +\infty$ and integer $j \geqslant 2$ equation (1) has a solution defined on the segment $[a, b]$, vanishing on its end points a, b and having exactly j zeros on $[a, b]$.*

The idea of proof is as follows: a solution $y(t)$ is oscillating if the conditions $y(a) = 0$, $y'(a) > 0$, ... , $y^{(n-1)}(a) > 0$ holds. We cannot use continuous dependence on parameters theorem ([6]), in the case $k \in (0, 1)$, because the theorem condition do not fulfill. Nevertheless, we prove continuous dependence on initial data in case of singular nonlinearity except the case $y(a) = 0$, $y'(a) = 0$, ... , $y^{(n-1)}(a) = 0$. If $y(a) = 0$, $y'(a) > 0$, ... , $y^{(n-1)}(a) > 0$ then we prove that the location of the N-th zero of solution $y(t)$ is changing continuously on initial data. Then we can obtain upper and lower estimates of this location. Finally we prove that there exist initial data such that the solution $y(t)$ with these initial data has the N-th zero exactly at the point b.

References

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