On Oscillatory Solutions of Second Order Nonlinear Delay Differential Equations

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In the interval $R_+ = [0, +\infty[$, the differential equation

$$u''(t) = f\big(t, u(\tau(t))\big) \tag{1}$$

is considered, where $f : R_+ \times R \to R$ and $\tau : R_+ \to R$ are continuous functions,

$$\tau(t) < t \ \text{ for } \ t \in R_+, \quad \lim_{t\to+\infty} \tau(t) = +\infty.$$

A continuous function $u : R_+ \to R$ is said to be **a solution of the differential equation (1)** if for some sufficiently large $a > 0$ it is twice continuously differentiable in the interval $]a, +\infty[$ and in this interval satisfies equation (1).

A solution u of equation (1) is said to be **proper** if for any positive number s there exists $t > s$ such that $u(t) \neq 0$.

A proper solution of equation (1) is said to be **oscillatory** if it has a sequence of zeros tending to $+\infty$, and otherwise it is said to be **nonoscillatory**.

Conditions are found under which equation (1) has only oscillatoty and unbounded nonoscillatory solutions.

In particular, the following theorem is proved.

Theorem. *Let*

$$f(t, x) \operatorname{sgn} x \geq \varphi(t, x) \operatorname{sgn} x > 0 \ \text{ for } \ t > 0, \ \ x \neq 0,$$

where $\varphi : R_+ \times R \to R$ is a nondecreasing in the second argument continuous function such that

$$\liminf_{t\to+\infty,\, x\to 0} \int\limits_{\tau^*(t)}^{t} (s - \tau^*(s)) \frac{\varphi(s, x)}{x}\, ds > 1,$$

where $\tau^*(t) = \max\{\tau(s) : \ 0 \leq s \leq t\}$. Then every proper solution of equation (1) *is either oscillatory or satisfying the condition*

$$\lim_{t\to+\infty} |u(t)| = +\infty. \tag{2}$$

At the same time, equation (1) *has both oscillatory proper solutions and monotone for large t solutions satisfying condition* (2).

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