

Solvability of the initial-boundary value problem of the Navier-Stokes equations of an inhomogeneous incompressible fluid in the half space

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Abstract: We are concerned with the global (in time) well-posedness issue for the initial boundary value problem pertaining to the inhomogeneous incompressible Navier-Stokes equations to a nonhomogeneous initial and boundary values.

There are lots of literature on the global well posedness when the boundary data of the velocity is given by zero values.

Nonzero boundary values of the velocity makes it difficult to be solved when the density is inhomogeneous.

When the initial density is close to a positive constant, the system can be reduced to Stokes type equations with nonzero initial and boundary values and transport equation with inflow boundary condition.

First, the transport equation with inflow boundary condition will be approximated by parabolic equation with Robin boundary condition, and we obtain strong solution of the approximate system for smooth enough data. Second, we construct a weak solution of the original nonlinear problem via Aubin-Lions compactness theorem, and by the properties of the renormalized solution of the transport equation.